Indices

1i $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

ii $9^0 = 1$

2i $(5a^2b)^3 \times 2b^4 = 125a^6b^3 \times 2b^4 = 250a^6b^7$

ii $\left(\frac{1}{16}\right)^{-1} = \left(\frac{16}{1}\right)^1 = 16$ (negative index is the reciprocal)

3 $\frac{(3xy^4)^3}{6x^5y^2} = \frac{27x^3y^{12}}{6x^5y^2} = \frac{9y^{10}}{2x^2}$ or $\frac{9}{2}x^{-2}y^{10}$

4i $\left(\frac{1}{4}\right)^0 = 1$

ii $16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{\sqrt{16}^3} = \frac{1}{4^3} = \frac{1}{64}$

5 $\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5 = 32$

6 $\left(\frac{1}{25}\right)^{-\frac{1}{2}} = \left(\frac{25}{1}\right)^{\frac{1}{2}} = \sqrt{25} = 5$

7i $25^{\frac{3}{2}} = \sqrt{25}^3 = 5^3 = 125$

ii $\left(\frac{7}{3}\right)^{-2} = \left(\frac{3}{7}\right)^2 = \frac{9}{49}$

8i $\left(\frac{9}{16}\right)^{-\frac{1}{2}} = \left(\frac{16}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$

ii $\frac{(2ac^2)^3 \times 9a^2c}{36a^4c^{12}} = \frac{8a^3c^6 \times 9a^2c}{36a^4c^{12}} = \frac{2a}{c^5}$ or $2ac^{-5}$

9i $125\sqrt{5} = 5^3 \times 5^{\frac{1}{2}} = 5^{\frac{7}{2}}$

ii $(4a^3b^5)^2 = 16a^6b^{10}$

Surds

1i $\frac{\sqrt{48}}{2\sqrt{27}} = \frac{\sqrt{16}\sqrt{3}}{2\sqrt{9}\sqrt{3}} = \frac{4\sqrt{3}}{2\times 3\sqrt{3}} = \frac{2}{3}$

ii $(5-3\sqrt{2})^2 = (5-3\sqrt{2})(5-3\sqrt{2}) = 25-15\sqrt{2}-15\sqrt{2}+9\times 2 = 43-30\sqrt{2}$

$3\sqrt{2}\times 3\sqrt{2} = 3\times 3\times\sqrt{2}\times\sqrt{2} = 9\times 2$

2i $\sqrt{75}+\sqrt{48} = \sqrt{25}\sqrt{3}+\sqrt{16}\sqrt{3} = 5\sqrt{3}+4\sqrt{3} = 9\sqrt{3}$

ii $\frac{14}{3-\sqrt{2}} = \frac{14(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{42+14\sqrt{2}}{9-2} = \frac{42+14\sqrt{2}}{7} = 6+2\sqrt{2}$

3i $\sqrt{98}-\sqrt{50} = \sqrt{49}\sqrt{2}-\sqrt{25}\sqrt{2} = 7\sqrt{2}-5\sqrt{2} = 2\sqrt{2}$

ii $\frac{6\sqrt{5}}{2+\sqrt{5}} = \frac{6\sqrt{5}(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} = \frac{12\sqrt{5}-6\times 5}{4-5} = \frac{-30+12\sqrt{5}}{-1} = 30-12\sqrt{5}$

4i $\sqrt{48}+\sqrt{27} = \sqrt{16}\sqrt{3}+\sqrt{9}\sqrt{3} = 4\sqrt{3}+3\sqrt{3} = 7\sqrt{3}$

ii $\frac{5\sqrt{2}}{3-\sqrt{2}} = \frac{5\sqrt{2}(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{15\sqrt{2}+5\times 2}{9-2} = \frac{10+15\sqrt{2}}{7}$

5i $\frac{1}{5+\sqrt{3}} = \frac{1\ (5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})} = \frac{5-\sqrt{3}}{25-3} = \frac{5-\sqrt{3}}{22}$

ii $(3-2\sqrt{7})^2 = (3-2\sqrt{7})(3-2\sqrt{7}) = 9-6\sqrt{7}-6\sqrt{7}+4\times 7 = 37-12\sqrt{7}$

6 $a+b+c = \frac{3}{2}+\frac{9-\sqrt{17}}{4}+\frac{9}{4} = \frac{6+9-\sqrt{17}+9+\sqrt{17}}{4} = \frac{24}{4} = 6$

$abc = \frac{3}{2}\left(\frac{9-\sqrt{17}}{4}\right)\left(\frac{9+\sqrt{17}}{4}\right) = \frac{3}{2}\left(\frac{81-17}{16}\right) = \frac{3}{2}\times\frac{64}{16} = 6$

$\therefore a+b+c = abc = 6$ QED

Algebraic fractions

1 $x^2 - 4 = (x+2)(x-2)$

$x^2 - 5x + 6 = (x-2)(x-3)$

$$\frac{x^2-4}{x^2-5x+6} = \frac{(x+2)(x-2)}{(x-2)(x-3)} = \frac{x+2}{x-3}$$

2 $3x^2 - 7x + 4 = 3x^2 - 3x - 4x + 4$
$= 3x(x-1) - 4(x-1)$
$= (3x-4)(x-1)$

$x^2 - 1 = (x+1)(x-1)$

$$\frac{3x^2-7x+4}{x^2-1} = \frac{(3x-4)(x-1)}{(x+1)(x-1)} = \frac{3x-4}{x+1}$$

Proof

1 $n^2+n = n(n+1)$ **or** n^2+n

when n is odd, $n+1$ is even
odd × even = even
when n is even, $n+1$ is odd
even × odd = even QED

when n is odd, n^2 is odd
odd + odd = even
when n is even, n^2 is even
even + even = even QED

2 $n^3-n = n(n^2-1)$ **or** n^3-n

when n is odd, n^2 is odd
n^2-1 is even
odd × even = even
when n is even, n^2 is even
n^2-1 is odd
even × odd = even QED

when n is odd, n^3 is odd
odd − odd = even
when n is even, n^3 is even
even − even = even QED

3i n is even, let $n = 2m$

$$
\begin{aligned}
3n^2+6n &= 3(2m)^2+6(2m) \\
&= 12m^2+12m \\
&= 12(m^2+m)
\end{aligned}
$$

∴ 12 is a factor when n is even QED

ii try n is odd, let $n = 2m+1$

$$
\begin{aligned}
3n^2+6n &= 3(2m+1)^2+6(2m+1) \\
&= 3(4m^2+4m+1)+6(2m+1) \\
&= 12m^2+12m+3+12m+6 \\
&= 12m^2+24m+9 \\
&= 12(m^2+2m)+9
\end{aligned}
$$

∴ 12 is not a factor when n is odd

4
$$
\begin{aligned}
n^3+3n^2+2n &= n(n^2+3n+2) \\
&= n(n+1)(n+2)
\end{aligned}
$$

this is the product of 3 consecutive numbers
- at least one must be even
- one must be a multiple of 3

a multiple of 2 × a multiple of 3 = a multiple of 6 QED

Solving linear inequalities

1. $6(x+3) > 2x+5$
$$6x+18 > 2x+5$$
$$4x > -13$$
$$x > -\frac{13}{4}$$

2. $3x-1 > 5-x$
$$4x > 6$$
$$x > \frac{3}{2}$$

3. $\frac{5x-3}{2} < x+5$
$$5x-3 < 2x+10$$
$$3x < 13$$
$$x < \frac{13}{3}$$

4. $\frac{3(2x+1)}{4} > -6$
$$3(2x+1) > -24$$
$$6x+3 > -24$$
$$6x > -27$$
$$x > -\frac{9}{2}$$

5. $7-x < 5x-2$
$$-6x < -9$$
$$6x > 9$$
$$x > \frac{3}{2}$$

6. $1-2x < 4+3x$
$$-5x < 3$$
$$5x > -3$$
$$x > -\frac{3}{5}$$

Solving equations

1. $$\frac{4x+5}{2x} = -3$$
$$4x+5 = -6x$$
$$10x = -5$$
$$x = -\tfrac{1}{2}$$

2. $$\frac{3x+1}{2x} = 4$$
$$3x+1 = 8x$$
$$5x = 1$$
$$x = \tfrac{1}{5}$$

3. $$y^2 - 7y + 12 = 0$$
$$(y-3)(y-4) = 0$$

$\therefore y = 3$ or $y = 4$

$$x^4 - 7x^2 + 12 = 0$$
$$(x^2)^2 - 7(x^2) + 12 = 0$$

Let $y = x^2$

$$y^2 - 7y + 12 = 0$$

$\therefore y = 3$ or $y = 4$
$\therefore x^2 = 3$ or $x^2 = 4$
$x = \pm\sqrt{3}$ or $x = \pm 2$

4. $$4x^2 + 20x + 25 = 0$$
$$4x^2 + 10x + 10x + 25 = 0$$
$$2x(2x+5) + 5(2x+5) = 0$$
$$(2x+5)(2x+5) = 0$$

$\therefore x = -\frac{5}{2}$

5. $$2x^2 + 3x = 0$$
$$x(2x+3) = 0$$

$\therefore x = 0$ or $x = -\frac{3}{2}$

Forming and solving equations

1i Area of triangle $= \frac{bh}{2}$

$$= \frac{(2x-3)(x+1)}{2} = 9$$

$$(2x-3)(x+1) = 18$$

$$2x^2 - x - 3 = 18$$

$$2x^2 - x - 21 = 0 \quad \text{QED}$$

ii $2x^2 - x - 21 = 0$

$2x^2 + 6x - 7x - 21 = 0$

$2x(x+3) - 7(x+3) = 0$

$(2x-7)(x+3) = 0$

$\therefore x = \frac{7}{2}$ or $x = -3$

this would give negative lengths $\therefore$ not possible

height $= x + 1 = \frac{7}{2} + 1 = 4{\cdot}5$ cm

base $= 2x - 3 = 2 \times \frac{7}{2} - 3 = 4$ cm

2i Area of trapezium $= \frac{(a+b)h}{2}$

$$= \frac{(x+2)+(3x+6)}{2} \times 2x = 140$$

$$\left(\frac{4x+8}{2}\right)2x = 140$$

$$4x^2 + 8x = 140$$

$$4x^2 + 8x - 140 = 0$$

$$x^2 + 2x - 35 = 0 \quad \text{QED}$$

ii $x^2 + 2x - 35 = 0$

$(x-5)(x+7) = 0$

$\therefore x = 5$ or $x = -7$

this would give negative lengths $\therefore$ not possible

$AB = 3x + 6 = 3 \times 5 + 6 = 21$ cm

Completing the square and turning points

1i $\underbrace{x^2+6x}+5$

$= (x+3)^2 - 9 + 5$
$= (x+3)^2 - 4$

$(x+3)^2 = x^2+6x+9$
$(x+3)^2 - 9 = x^2+6x$

ii coordinates of minimum point $= (-3, -4)$

2i $\underbrace{x^2-7x}+6$

$= (x-\frac{7}{2})^2 - \frac{49}{4} + 6$
$= (x-\frac{7}{2})^2 - \frac{25}{4}$

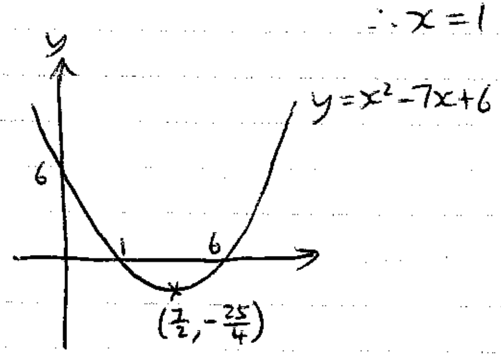
$(x-\frac{7}{2})^2 = x^2 - 7x + \frac{49}{4}$
$(x-\frac{7}{2})^2 - \frac{49}{4} = x^2 - 7x$

ii coordinates of minimum point $= (\frac{7}{2}, -\frac{25}{4})$

iii crosses y-axis at $y=6$

crosses x-axis when $x^2-7x+6=0$
$(x-1)(x-6)=0$

$\therefore x=1$ or $x=6$



3i $3x^2+6x+10$
$= 3(\underbrace{x^2+2x}) + 10$
$= 3[(x+1)^2 - 1] + 10$
$= 3(x+1)^2 - 3 + 10$
$= 3(x+1)^2 + 7$

$(x+1)^2 = x^2+2x+1$
$(x+1)^2 - 1 = x^2+2x$

ii minimum value of $y=7$
$\therefore$ always above x-axis QED

4i $\quad 4x^2-24x+27$

$= 4(\underbrace{x^2-6x}) + 27$

$= 4[(x-3)^2-9] + 27$

$= 4(x-3)^2 - 36 + 27$

$= 4(x-3)^2 - 9$

$(x-3)^2 = x^2-6x+9$

$(x-3)^2-9 = x^2-6x$

ii coordinates of minimum point $= (3,-9)$

iii $4x^2-24x+27=0$

$4(x-3)^2-9 = 0$

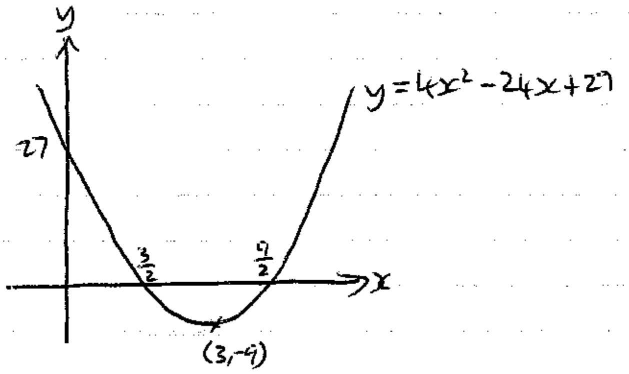
$4(x-3)^2 = 9$

$(x-3)^2 = \frac{9}{4}$

$x-3 = \pm\frac{3}{2}$

$x = 3\pm\frac{3}{2}$

$\therefore x = \frac{3}{2}$ or $x = \frac{9}{2}$



Discriminant and roots

1. $b^2 - 4ac = (5)^2 - 4(3)(2)$
$= 1$

$\therefore$ 2 distinct real roots

2 no real roots

$$\begin{aligned} \therefore \quad b^2 - 4ac &< 0 \\ (3)^2 - 4(2)(-k) &< 0 \\ 9 + 8k &< 0 \\ 8k &< -9 \\ k &< -\tfrac{9}{8} \end{aligned}$$

3 no real roots

$$\begin{aligned} \therefore \quad b^2 - 4ac &< 0 \\ k^2 - 4(2)(2) &< 0 \\ k^2 - 16 &< 0 \\ k^2 &< 16 \\ -4 < k &< 4 \end{aligned}$$

4 for intersection, solve simultaneously

$$x^2 - 5x + 7 = 3x - 10$$
$$x^2 - 8x + 17 = 0$$

$$b^2 - 4ac = (-8)^2 - 4(1)(17) = -4 < 0$$

$\therefore$ no solution
$\therefore$ lines do not intersect QED

Changing the subject of a formula

1
$$s = ut + \tfrac{1}{2}at^2$$
$$\tfrac{1}{2}at^2 = s - ut$$
$$a = \frac{2(s-ut)}{t^2}$$

2
$$V = \tfrac{1}{3}\pi r^2 h$$
$$r^2 = \frac{3V}{\pi h}$$
$$r = \pm\sqrt{\frac{3V}{\pi h}}$$

3
$$a = \frac{\sqrt{y} - 5}{c}$$
$$\sqrt{y} - 5 = ac$$
$$\sqrt{y} = ac + 5$$
$$y = (ac+5)^2$$

4
$$c = \sqrt{\frac{a+b}{2}}$$
$$\frac{a+b}{2} = c^2$$
$$a + b = 2c^2$$
$$a = 2c^2 - b$$

5
$$V = \tfrac{1}{3}\pi r^2 \sqrt{l^2 - r^2}$$
$$\sqrt{l^2 - r^2} = \frac{3V}{\pi r^2}$$
$$l^2 - r^2 = \left(\frac{3V}{\pi r^2}\right)^2$$
$$l^2 = \left(\frac{3V}{\pi r^2}\right)^2 + r^2$$
$$l = \pm\sqrt{\left(\frac{3V}{\pi r^2} + r^2\right.}$$

continued →

6 $2a+5c=af+7c$

$2a-af=7c-5c$

$a(2-f)=2c$

$a=\dfrac{2c}{2-f}$

7 $y+5=x(y+2)$

$y+5=xy+2x$

$y-xy=2x-5$ or $xy-y=5-2x$

$y(1-x)=2x-5$ $\quad y(x-1)=5-2x$

$y=\dfrac{2x-5}{1-x}$ $\quad y=\dfrac{5-2x}{x-1}$

8 $P=\dfrac{C}{C+4}$

$P(C+4)=C$

$PC+4P=C$

$C-PC=4P$ or $PC-C=-4P$

$C(1-P)=4P$ $\quad C(P-1)=-4P$

$C=\dfrac{4P}{1-P}$ $\quad C=-\dfrac{4P}{P-1}$

9 $y=\dfrac{1-2x}{x+3}$

$y(x+3)=1-2x$

$xy+3y=1-2x$

$xy+2x=1-3y$

$x(y-2)=1-3y$

$x=\dfrac{1-3y}{y-2}$

Equation of a straight line

1 $y = 5x - 4$, parallel line $y = 5x + c$ when $x = 2, y = 13$

$$13 = 5 \times 2 + c$$
$$c = 3$$
$$\therefore y = 5x + 3$$

2 $y = 3x + 1$, parallel line $y = 3x + c$ when $x = 4, y = 5$

$$5 = 3 \times 4 + c$$
$$c = -7$$
$$\therefore y = 3x - 7$$

3 gradient, $m = \frac{11 - -9}{3 - -1} = \frac{20}{4} = 5$

$y = 5x + c$ when $x = 3, y = 11$

$$11 = 5 \times 3 + c$$
$$c = -4$$
$$\therefore y = 5x - 4$$

4 $3x + 2y = 6$

$$2y = -3x + 6$$

$y = -\frac{3}{2}x + 3$, parallel line $y = -\frac{3}{2}x + c$ when $x = 2, y = 10$

$$10 = -\frac{3}{2} \times 2 + c$$
$$c = 13$$

$\therefore y = -\frac{3}{2}x + 13$ or $3x + 2y = 26$

5 i gradient, $m = \frac{9 - 1}{3 - -1} = \frac{8}{4} = 2$

$y = 2x + c$ when $x = 3, y = 9$

$$9 = 2 \times 3 + c$$
$$c = 3$$
$$\therefore y = 2x + 3$$

ii gradient, $-\frac{1}{m} = -\frac{1}{2}$

coordinates of midpoint $= \left(\frac{-1+3}{2}, \frac{1+9}{2}\right) = (1, 5)$

$y = -\frac{1}{2}x + c$ when $x = 1, y = 5$

$$5 = -\frac{1}{2} \times 1 + c$$
$$c = \frac{11}{2}$$
$$\therefore y = -\frac{1}{2}x + \frac{11}{2}$$
$$2y + x = 11$$

QED

Intersection of two lines

1. $y = 3x + 1$ —— ①
   $x + 3y = 6$ —— ②

   sub. ① into ②

   $$x + 3(3x+1) = 6$$
   $$x + 9x + 3 = 6$$
   $$10x = 3$$
   $$x = \frac{3}{10}$$

   sub. $x = \frac{3}{10}$ into ①

   $$y = 3 \times \frac{3}{10} + 1$$
   $$= \frac{19}{10}$$

   coordinates of point of intersection $= \left(\frac{3}{10}, \frac{19}{10}\right)$

2. $y = 2x - 5$ —— ①
   $6x + 2y = 7$ —— ②

   sub. ① into ②

   $$6x + 2(2x - 5) = 7$$
   $$6x + 4x - 10 = 7$$
   $$10x = 17$$
   $$x = \frac{17}{10}$$

   sub. $x = \frac{17}{10}$ into ①

   $$y = 2 \times \frac{17}{10} - 5$$
   $$= -\frac{8}{5}$$

   coordinates of point of intersection $= \left(\frac{17}{10}, -\frac{8}{5}\right)$

3 $y = x^2 - 6x + 2$ —— ①
$y = 2x - 14$ —— ②

equate ① and ②

$x^2 - 6x + 2 = 2x - 14$
$x^2 - 8x + 16 = 0$
$(x-4)^2 = 0$

$\therefore x = 4$

sub. $x=4$ into ②

$y = 2\times 4 - 14$
$= -6$

only one point of intersection $(4,-6)$
$\therefore$ $y=2x-14$ is a tangent to $y=x^2-6x+2$ QED

4 $y = 4x^2 + 24x + 31$ —— ①
$x + y = 10$ —— ②

from ②, $y = 10 - x$ —— ③

equate ① and ③

$\therefore 4x^2 + 24x + 31 = 10 - x$
$4x^2 + 25x + 21 = 0$
$4x^2 + 4x + 21x + 21 = 0$
$4x(x+1) + 21(x+1) = 0$
$(4x+21)(x+1) = 0$

$\therefore x = -\frac{21}{4}$ or $x = -1$

sub. $x = -\frac{21}{4}$ into ③

$y = 10 - -\frac{21}{4} = \frac{61}{4}$
$= \frac{61}{4}$

sub. $x=-1$ into ③

$y = 10 - -1$
$= 11$

coordinates of point of intersection $= (-\frac{21}{4}, \frac{61}{4})$ and $(-1, 11)$

5. $x^2+y^2=25$ —— ①
$y=3x$ —— ②

sub. ② into ①

$$x^2+(3x)^2=25$$
$$x^2+9x^2=25$$
$$10x^2=25$$
$$x^2=\frac{5}{2}$$
$$x=\pm\sqrt{\frac{5}{2}}=\pm\frac{\sqrt{10}}{2}$$

$$\sqrt{\frac{5}{2}}=\frac{\sqrt{5}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{10}}{2}$$

sub. $x=-\frac{\sqrt{10}}{2}$ into ②

$$y=-\frac{3\sqrt{10}}{2}$$

sub. $x=\frac{\sqrt{10}}{2}$ into ②

$$y=\frac{3\sqrt{10}}{2}$$

coordinates of point of intersection $=\left(-\frac{\sqrt{10}}{2},-\frac{3\sqrt{10}}{2}\right)$ and $\left(\frac{\sqrt{10}}{2},\frac{3\sqrt{10}}{2}\right)$

6i. $x^2+y^2=45$ —— ①

centre $=(0,0)$ and radius $=\sqrt{45}=3\sqrt{5}$

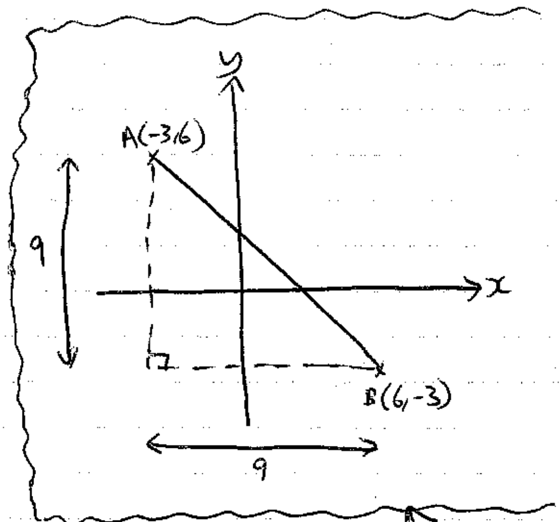
ii. $x+y=3$ —— ②

from ②, $y=3-x$ —— ③

sub. ③ into ①

$$x^2+(3-x)^2=45$$
$$x^2+x^2-6x+9=45$$
$$2x^2-6x-36=0$$
$$x^2-3x-18=0$$
$$(x+3)(x-6)=0$$

$\therefore x=-3$ or $x=6$



sub $x=-3$ into ③

$$y=3--3$$
$$=6$$

sub. $x=6$ into ③

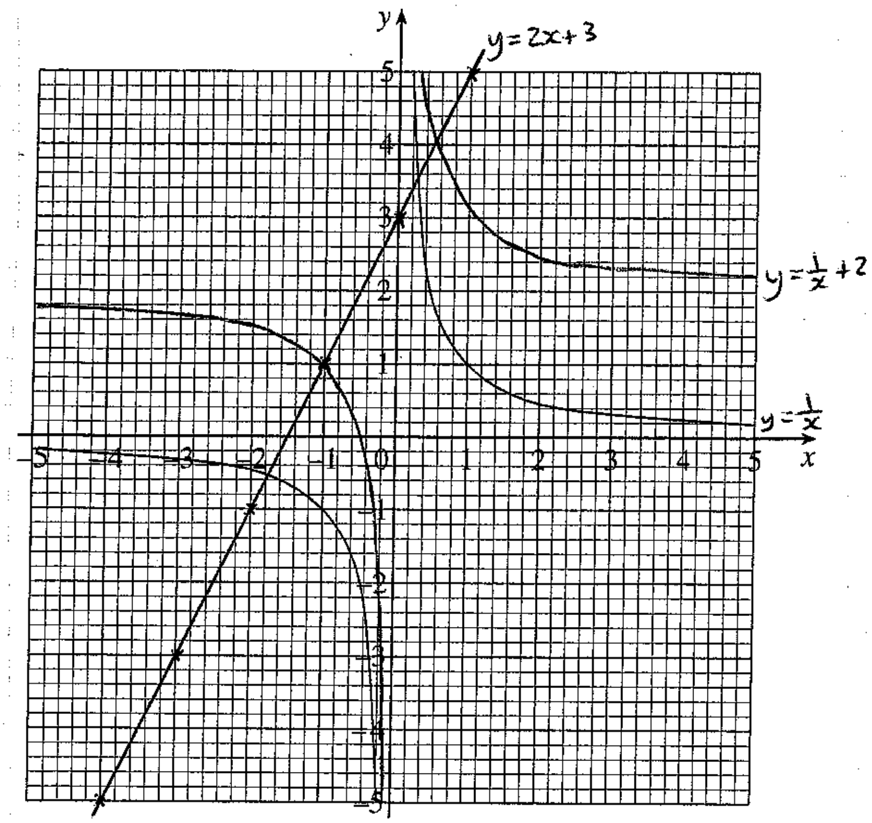
$$y=3-6$$
$$=-3$$

coordinates of point of intersection $=(-3,6)$ and $(6,-3)$

$AB^2=9^2+9^2$
$AB=\sqrt{162}$ QED

Using graphs to solve equations

1



i $\frac{1}{x} = 2x + 3$

intersection of $y = \frac{1}{x}$ and $y = 2x + 3$

$x \approx -1.8$ or $x \approx 0.3$

ii $\frac{1}{x} = 2x + 3$

$1 = 2x^2 + 3x$

$2x^2 + 3x - 1 = 0$

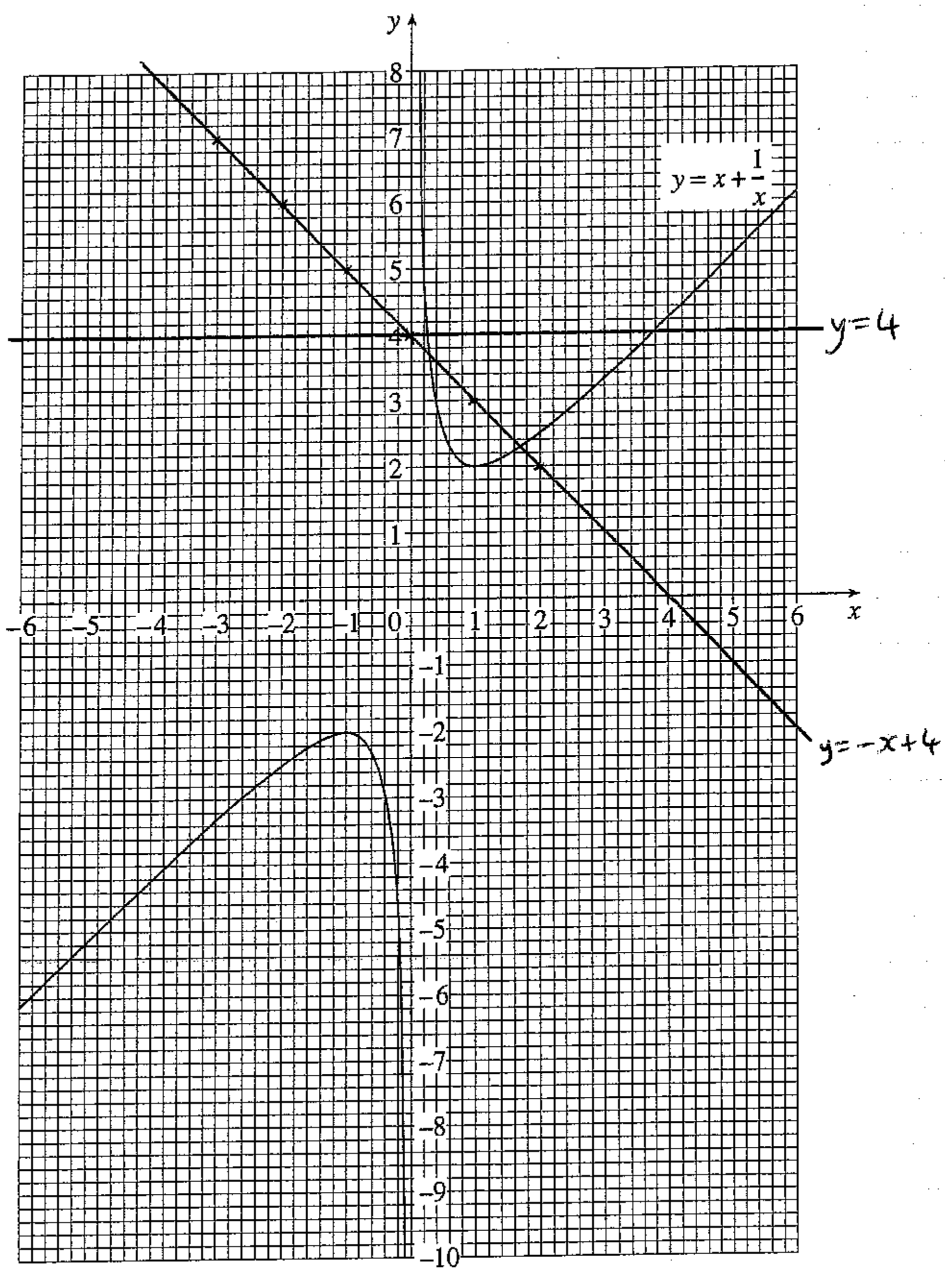
$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{17}}{4}$$

iii $y = \frac{1}{x} + 2$

translate graph of $y = \frac{1}{x}$ by vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

2



A. $x + \frac{1}{x} = 4$

intersection of $y = x + \frac{1}{x}$ and $y = 4$

$x \approx 0.3$ or $x \approx 3.7$

B. $2x + \frac{1}{x} = 4$

$x + \frac{1}{x} = -x + 4$

intersection of $y = x + \frac{1}{x}$ and $y = -x + 4$

$x \approx 0.3$ or $x \approx 1.7$

Transformation of graphs

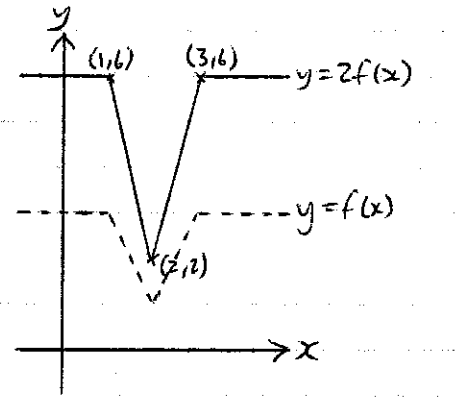
1i $(10, 4)$

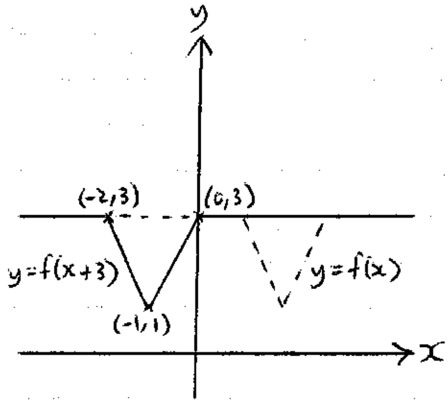
ii $(5, 11)$

2i $(3, 15)$

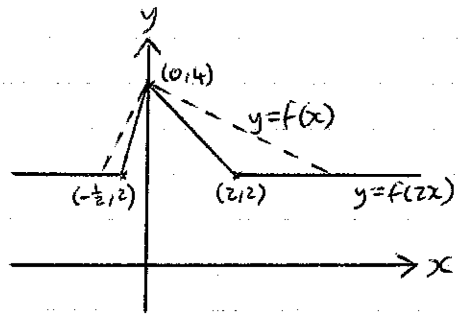
ii $(\frac{3}{2}, 5)$

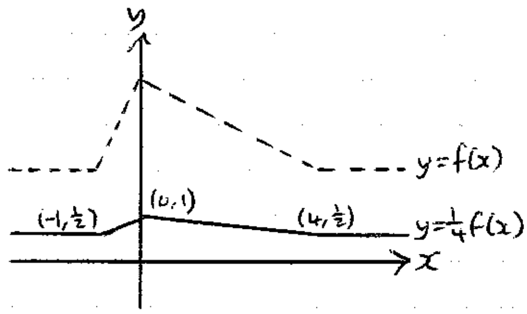
3 i





4 i





5i $y = 2f(x)$

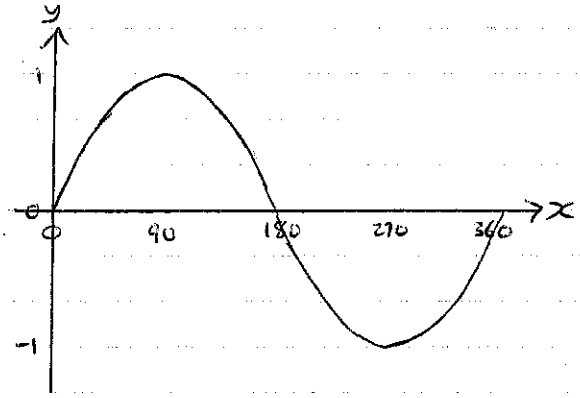
ii $y = f(x-3)$

6 $y = (x-2)^2 - 4$

7 translation by vector $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

Trigonometric graphs

1



$\sin x = -0.68$

$x = \sin^{-1}(-0.68)$

$= -42.8°$

or $x = 180 - (-42.8)$ by symmetry

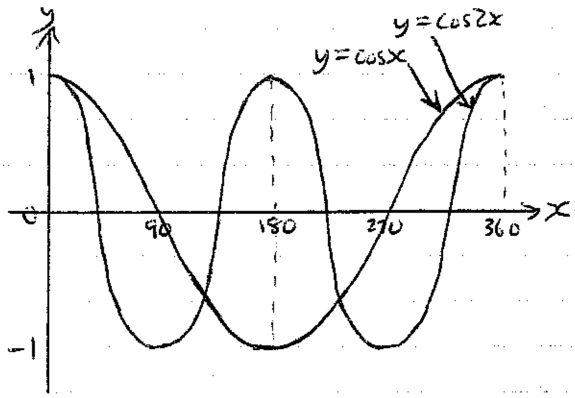
$= 222.8°$

or $x = -42.8 + 360$ periodic $(360°)$

$= 317.2°$

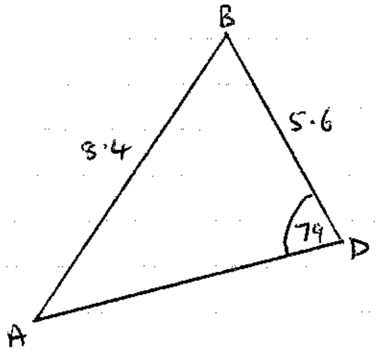
$\therefore x = 222.8°$ or $317.2°$

2.



Sine rule and cosine rule

1i

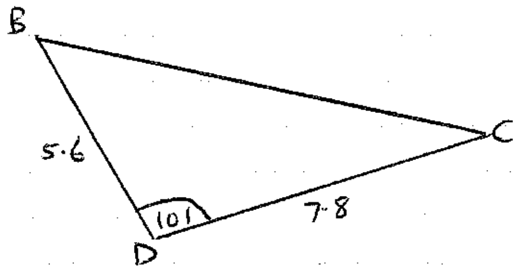


$$\frac{\sin BAD}{5\cdot 6} = \frac{\sin 79}{8\cdot 4}$$

$$BAD = \sin^{-1}\left(5\cdot 6 \frac{\sin 79}{8\cdot 4}\right)$$

$$= 40\cdot 9^\circ$$

ii

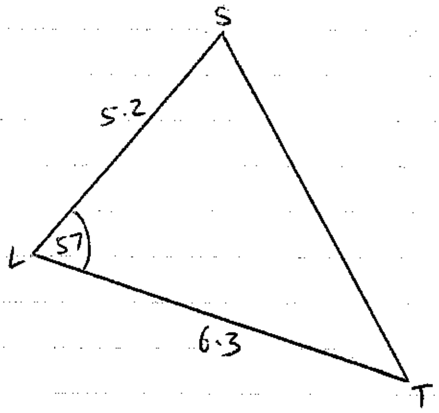


$$BC^2 = (5\cdot 6)^2 + (7\cdot 8)^2 - 2(5\cdot 6)(7\cdot 8)\cos 101^\circ$$

$BC = 10\cdot 4$ cm

continued →

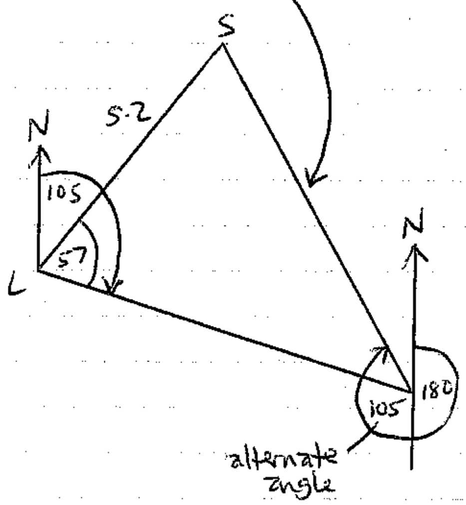
2i



$ST^2 = (5.2)^2 + (6.3)^2 - 2(5.2)(6.3)\cos 57°$

$ST = 5.6$ km

ii



$$\frac{\sin LTS}{5.2} = \frac{\sin 57}{5.5718\ldots}$$

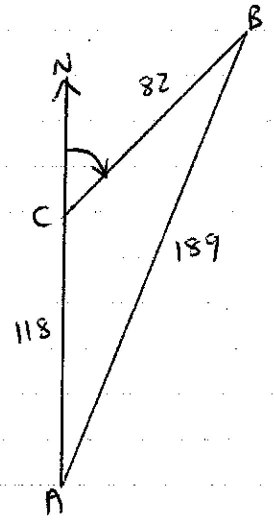
$$LTS = \sin^{-1}\left(5.2\,\frac{\sin 57}{5.5718\ldots}\right)$$

$$= 51.5°$$

bearing of S from T $= 180 + 105 + 51.5$

$= 336.5°$ or $337°$

3i



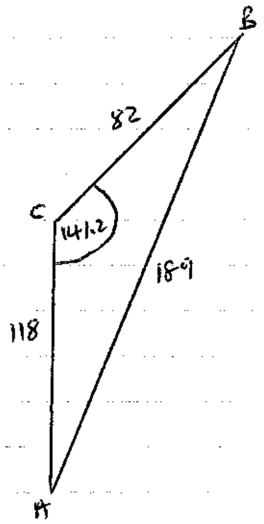
$$(189)^2 = (82)^2 + (118)^2 - 2(82)(118)\cos ACB$$

$$\cos ACB = \frac{(82)^2 + (118)^2 - (189)^2}{2(82)(118)}$$

$$ACB = 141.2^\circ$$

bearing of B from C $= 180 - 141.2$
$= 038.8^\circ$ or 039°

ii



area $= \frac{1}{2}ab\sin C$
$= \frac{1}{2}(82)(118)\sin 141.2$
$= 3034.2\ m^2$

Arc length and sector area

1 arc length $= 2\pi r \times \frac{\theta}{360}$

$$2\pi(18\cdot0) \times \frac{\theta}{360} = 43\cdot2$$

$$\theta = \frac{43\cdot2 \times 360}{2\pi(18\cdot0)}$$

$$= 137\cdot5^\circ$$

2 sector area $= \pi r^2 \times \frac{\theta}{360}$

$$\pi(5)^2 \times \frac{\theta}{360} = 9$$

$$\theta = \frac{9 \times 360}{\pi(5)^2}$$

$$= 41\cdot3^\circ$$